Toward a Thermodynamic Framework for UNNS

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Abstract

We propose a thermodynamic framework for the UNNS (Unbounded Nested Number Sequence) substrate. By defining energy, entropy, and temperature of recursion, we establish connections between recursive dynamics and statistical mechanics. The theory introduces ensembles of nests, energy functionals based on echo residues and spectral constants, and formulates entropy production under operator-driven dynamics. This opens a pathway toward non-equilibrium statistical mechanics of recursion.

1 Introduction

The UNNS substrate treats recursion as a fundamental generator of structure. Previous work established operators (inlaying, inletting, repair, transsentifying) and constants. Here, we build a *thermodynamic layer*, introducing concepts of energy, entropy, and temperature for recursive nests.

The motivation is twofold:

- to provide *quantitative metrics* of stability, complexity, and order within recursion;
- to link UNNS to established frameworks in statistical mechanics, information theory, and complexity science.

2 Ensembles of Nests

Let $\mathcal{N}_k$ denote the set of UNNS nests truncated to depth k. Each nest $N \in \mathcal{N}_k$ is described by recurrence coefficients $\mathbf{a} = (a_0, \ldots, a_k)$, echo residues $\{e_n\}$, and spectral constants $\sigma(N)$.

Definition 2.1 (Energy functional)**.** *An* energy functional *is a map*

$$E : \mathcal{N}_k \to \mathbb{R}_{\geq 0}$$

measuring instability or complexity of a nest.

Examples include:

$$\begin{aligned} E_{\text{echo}}(N) &= \sum_{n=0}^{k} w_n |e_n|^2, \\ E_{\text{spec}}(N) &= \operatorname{dist}\big(\sigma(N), \mathcal{O}_K\big)^2, \\ E_{\text{comp}}(N) &= \ell(N), \end{aligned}$$

where $\ell(N)$ is description length and $\mathcal{O}_K$ an algebraic integer ring.

Definition 2.2 (UNNS canonical ensemble)**.** *For inverse temperature $\beta > 0$, the probability of $N \in \mathcal{N}_k$ is*

$$p_\beta(N) = \frac{1}{Z(\beta)} e^{-\beta E(N)}, \qquad Z(\beta) = \sum_{N \in \mathcal{N}_k} e^{-\beta E(N)}.$$

3 Thermodynamic Quantities

Definition 3.1 (Average energy)**.**

$$\langle E \rangle_\beta = \sum_N p_\beta(N) E(N).$$

Definition 3.2 (Entropy)**.** *The Gibbs entropy of the ensemble is*

$$S(\beta) = -\sum_N p_\beta(N) \log p_\beta(N) = \beta \langle E \rangle_\beta + \log Z(\beta).$$

Definition 3.3 (Free energy)**.**

$$F(\beta) = -\frac{1}{\beta} \log Z(\beta).$$

Proposition 3.1. *Entropy satisfies the thermodynamic identity*

$$\frac{\partial S}{\partial \langle E \rangle} = \frac{1}{T},$$

where $T = 1/\beta$ is the UNNS temperature.

4 Dynamics and Non-equilibrium

Recursive operators (inlaying, inletting, repair, etc.) act as stochastic or deterministic maps on $\mathcal{N}_k$. We can model their action as a Markov process with transition rates $W(N \to N')$.

Definition 4.1 (Entropy production rate)**.** *Let $P_t(N)$ be the time-dependent distribution. The entropy production rate is*

$$\sigma(t) = \frac{d}{dt}S(t) + \sum_{N,N'} J_{N\to N'}(t) \ln \frac{W(N \to N')}{W(N' \to N)},$$

where $J_{N\to N'}(t) = P_t(N)W(N \to N') - P_t(N')W(N' \to N)$.

Lemma 4.1 (Second law for UNNS)**.** *For any operator-driven evolution,*

$$\Delta S_{sys} + \Delta S_{env} \geq 0.$$

5 Worked Example: Inletting-driven Expansion

Consider the multiplicative recurrence

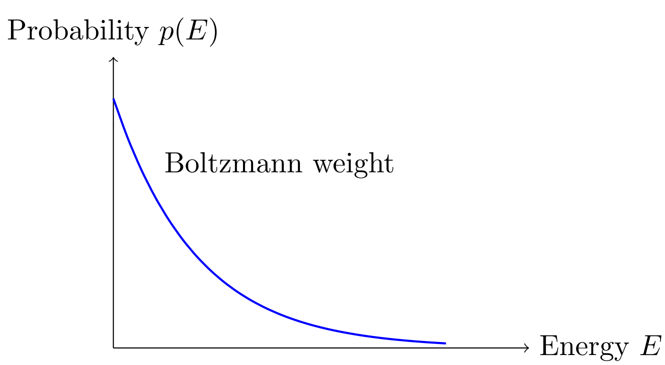
$$S_{k+1} = (1 + \alpha T_k)S_k,$$

with $T_k = T_0 + \eta_k$ where η_k are i.i.d. fluctuations. Define energy

$$E(N) = \sum_{n=0}^{k-1} \frac{\eta_n^2}{2\sigma^2}.$$

This yields Gaussian weights in the ensemble. Partition function factorizes, and entropy can be computed analytically.

6 Visualization



7 Discussion

This framework provides:

- metrics of complexity and stability of nests;
- a foundation for non-equilibrium analysis of operator actions;
- connections to statistical mechanics, complexity, and information theory.

8 Conclusion

UNNS thermodynamics quantifies recursion through analogs of energy, entropy, and temperature. Future directions include phase transition analysis, fluctuation theorems, and connections to cognitive and physical systems.

References

[1] J. W. Gibbs, *Elementary Principles in Statistical Mechanics*, 1902.

[2] R. Kubo, *The fluctuation-dissipation theorem*, Rep. Prog. Phys., 1966.

[3] T. Cover, J. Thomas, *Elements of Information Theory*, Wiley, 1991.